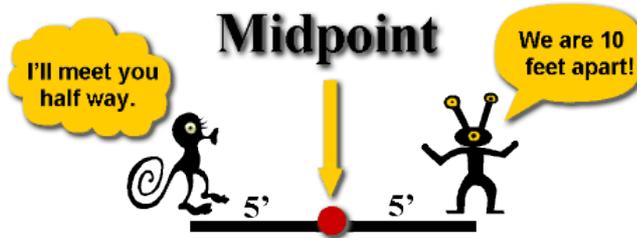# LESSON 8 ANALITIC GEOMETRY

It is impossible to be a mathematician without being a poet in soul. Sofia Kovalevskaya

1. RELATIONSHIPS BETWEEN POINTS IN THE PLANE
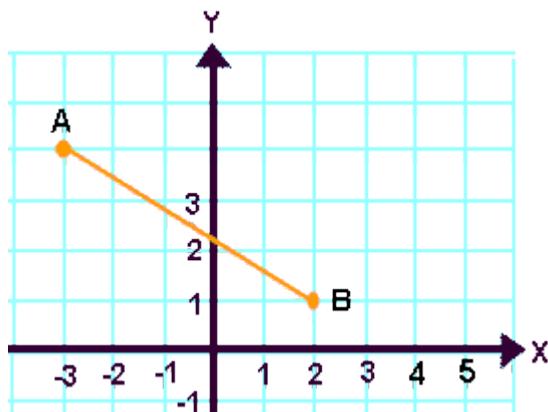
Midpoint of a line segment.
The point halfway between the endpoints of a line segment is called the **midpoint**. A midpoint divides a line segment into two equal segments.

The Midpoint Formula: The midpoint of a segment with endpoints (x_1 , y_1) and (x_2 , y_2) has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example:

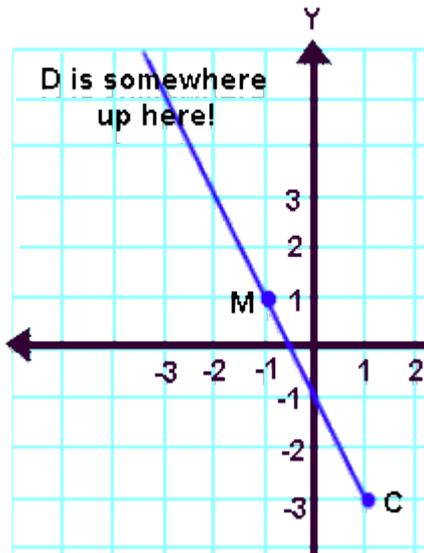**Find the midpoint of line segment $\overline{AB}$.**

$A(-3,4)$
$B(2,1)$
The midpoint will have coordinates

$$\left(\frac{-3+2}{2}, \frac{4+1}{2} \right)$$

$$\left(\frac{-1}{2}, \frac{5}{2} \right)$$

1

M is the midpoint of $\overline{CD}$. The coordinates M(-1,1) and C(1,-3) are given.
Find the coordinates of point D.

$M(-1,1)$, $C(1,-3)$ and $D(x,y)$
Substitute into the Midpoint Formula:

$$\left(-1,1\right) = \left(\frac{x+1}{2}, \frac{y+(-3)}{2}\right)$$

Solve for each variable separately:

$$\frac{x+1}{2} = -1 \qquad \frac{y+(-3)}{2} = 1$$

$$x+1 = -2 \qquad y+(-3) = 2$$

$$x = -3 \qquad\quad y = 5$$

Exercises:
1. Find the midpoint of the segment connecting the points (6,4) and (3,-4).
2. Find the midpoint of the segment connecting the points (a,b) and (3a,c).

3. *M* is the midpoint of $\overline{AB}$. The coordinates of *A* are (-2,3) and the coordinates of *M* are (1,0). Find the coordinates of *B*.
4. The coordinates of quadrilateral *ABCD* are A(-3,-1), B(3,1), C(7,5), and D(1,3). Do the diagonals bisect each other?

5. *M* is the midpoint of $\overline{AB}$. The coordinates of *A* are (2,3) and the coordinates of *M* are (4.5,6). Find the coordinates of *B*.

6. $\overline{CD}$ is the diameter of a circle whose center is the point (2,1). If the coordinates of *C* are (0,-2), find the coordinates of *D*.

2
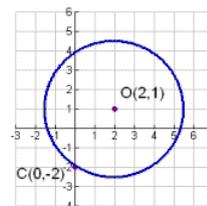
Three Collinear Points

Three points are collinear if they lay in the same line.
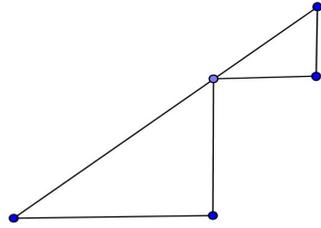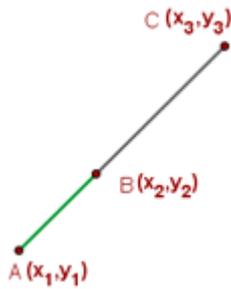
and thus, the sides are proportional:

$$\frac{x_2 - x_1}{x_3 - x_2} = \frac{y_2 - y_1}{y_3 - y_2}$$

If the three points are in the same line, the two triangles are similar

Examples:

1. Determine whether A = (−2, −3), B = (1, 0) and C = (6, 5) are alligned points.

$$\frac{1+2}{6-1} = \frac{0+3}{5-0} \qquad \frac{3}{5} = \frac{3}{5} \qquad Yes$$

2. Calculate the value of **a** in the following aligned points.

$$A = (2,\ 1) \quad B = (4,\ 2) \quad C = (6,\ a)$$

$$\frac{4-2}{6-4} = \frac{2-1}{a-2} \qquad a = 3$$

Exercises:
1. Determine whether R(2,7), S(5,1) y T(15,25) are alligned points.
2. Calculate the value of **a** in the following aligned points: A(2,7), B(5,-1) and C(a,-25).
3. Given A(0,1), B(2,5) and P(x,y), how must be **x** and **y** so that A, B and P are aligned?
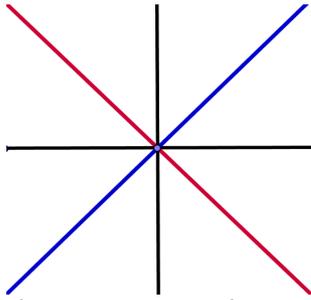4. Calculate the value of **t** so that A(1,2), B(7,-11) and C(t,2t) are aligned.

2. EQUATIONS OF LINES

Bisectors of quadrants.

The bisector of the first (and third) quadrant has the equation y = x, so its slope is m = 1. (blue line)
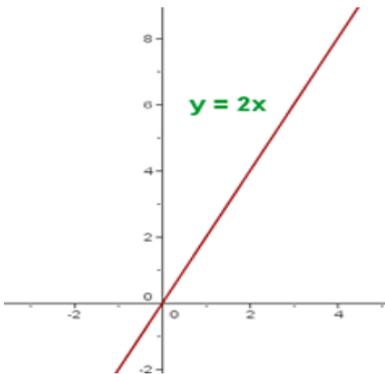
The bisector of the second (and fourth) quadrant has the equation y=-x, so its slope is m=-1. (red line)

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Other lines passing through the origin:

Their equation is **y=mx**

Slope Intercept Form:
[used when you know, or can find, the slope, *m*, and the *y*-intercept, *n*.]
y = mx + n

Point Slope Form:
[used when you know, or can find, a point on the line *(x₁, y₁)*, and the slope, *m*.]
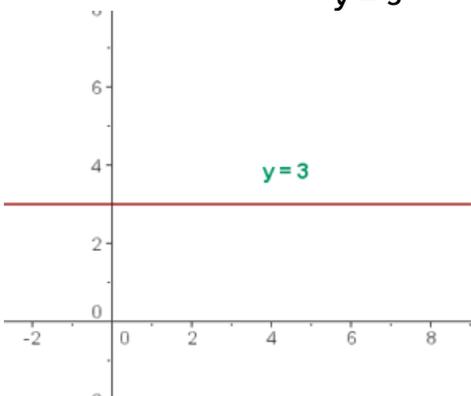
$$y - y_1 = m(x - x_1)$$

Horizontal Line Form:
y = 3 (or any number)
Lines that are horizontal have a slope of zero.
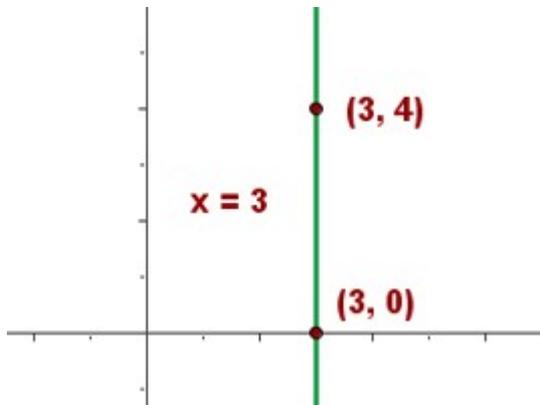
$$y = mx + b$$
$$y = 0x + 3$$
$$y = 3$$

4

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Vertical Line Form:
x = 3 (or any number)
Lines that are vertical have no slope (it does not exist).

Two point form:

If we know the coordinates of two points of the line $A(x_1, y_1)$ and $B(x_2, y_2)$ any other point $P(x,y)$, will be aligned so that:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

The two-point form equation of the line can also be written as:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Examples:

1. Find the slope and y-intercept for the equation $2y = -6x + 8$.

First solve for "y =": $y = -3x + 4$
Remember the form: $y = mx + b$
Answer: the slope (*m*) is -3
          the y-intercept (*b*) is 4

2. Given that the slope of a line is -3 and the line passes through the point (-2,4), write the equation of the line.

The slope: $m = -3$
The point $(x_1, y_1) = (-2,4)$
Remember the form: $y - y_1 = m (x - x_1)$
Substitute: $y - 4 = -3 (x - (-2))$
ANS. $y - 4 = -3 (x + 2)$

If asked to express the answer in "y =" form: $y - 4 = -3x - 6$
                                                  $y = -3x - 2$

3. Find the equation of the line whose slope is 4 and the coordinates of the *y*-intercept are (0,2).

5

In this problem *m* = 4 and *b* = 2.
Remember the form: *y* = *mx* + *b* and that *b* is where the line crosses the *y*-axis.
Substitute: *y* = 4*x* + 2

4. Find the slope of the line that passes through the points (-3,5) and (-5,-8).

$$\frac{x-(-3)}{-5-(-3)} = \frac{y-5}{-8-5} \rightarrow \frac{x+3}{-2} = \frac{y-5}{-13} \rightarrow$$

$$y-5 = \frac{-13(x+3)}{-2} \rightarrow y-5 = 6.5(x+3)$$

Exercises:

1. Which of the following equations passes through the points (2,4) and (-3,-6)?

Choose:

a) *y* = (1/2)*x* - 2   b) *y* = 2*x*   c) *y* = 2*x* + 4   d) *y* = (-1/2)*x* + 2

2. Which of the following equations passes through the points (2,1) and (5,-2)?
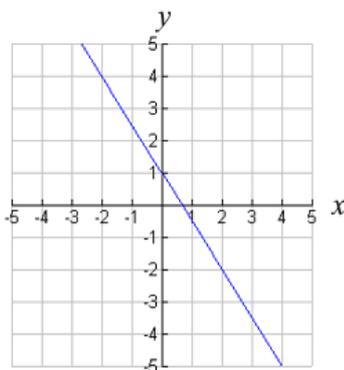
Choose:

a) $y = \frac{3}{7}x + 5$ b) *y* = -*x* + 3   c) *y* = -*x*+2   d) $y = -\frac{1}{3}x + 3$

2. Does the graph of the straight line with slope of -2 and *y*-intercept of -3 pass through the point (5,-13)?

Choose:

a) Yes   b) No

3. The slope of this line is 3/2.

Choose:

a) True
b) False

4. What is the slope of the line 3*x* + 2*y* = 12?
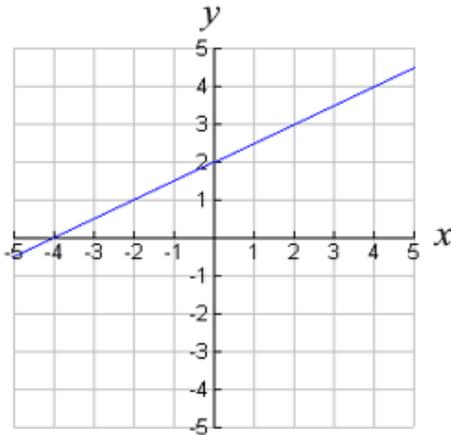
Choose:

a) 3   b) $\dfrac{3}{2}$ c) $-\dfrac{3}{2}$ d) 2

5. Which is the correct slope of the line through (-2,3) and (4,-5)?

Choose:

a) $-\dfrac{4}{3}$ b) $-\dfrac{3}{4}$ c) $\dfrac{4}{3}$ d) $-\dfrac{1}{3}$

6. The slope of this line is 1/2.

Choose:

a) True
b) False

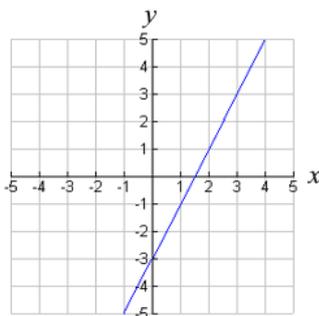7. Does the line $2y + x = 7$ pass through the point (1,3)?

Choose:

a) Yes   b) No

8. Which is the equation of a line that passes through the point (2,5) and has a slope of -3?

Choose:

a) $y = -3x - 3$ b) $y = -3x + 17$ c) $y = -3x + 11$ d) $y = -3x + 5$

9. What is the equation of this line?

Choose:

a) $y = \dfrac{1}{2}x - 3$ b) $y = 2x - 3$

c) $y = -\dfrac{1}{2}x - 3$ d) $y = -2x - 3$

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10. What is an equation for the line that passes through the coordinates (2,0) and (0,3)?

Choose:

a) $y = -\dfrac{3}{2}x + 3$ b) $y = -\dfrac{3}{2}x - 3$ c) $y = -\dfrac{2}{3}x + 2$ d) $y = -\dfrac{2}{3}x - 2$

More exercises here[1].

3. PARALLEL AND PERPENDICULAR LINES

Slope of parallel lines. Two parallel lines have the same slope. The symbol to indicate parallel lines is two vertical bars.

$l_1 \parallel l_2 \rightarrow m_1 = m_2$ where l_1 and l_2 are lines m_1 and m_2 are slopes.

$y = 3x + 5$ These lines are ALL parallel.
$y = 3x - 7$ They all have the same slope
$y = 3x + 0.5$ m=3
$y = 3x$

Slope of perpendicular lines. Perpendicular lines have negative reciprocal slopes. The symbol to indicate perpendicular is an up-side-down capital T.

$l_1 \perp l_2 \rightarrow m_1 = -\dfrac{1}{m_2}$ where l_1 and l_2 are lines m_1 and m_2 are slopes.

$y = 4x + 7$ These lines are perpendicular.
                        Their slopes (*m*) are negative
$y = -\dfrac{1}{4}x - 6$ reciprocals.

Exercises:

1. $l_1 \parallel l_2$. If the slope of l_1 is $\dfrac{x}{4}$, and the slope of l_2 is $\dfrac{x+5}{6}$, find the value of x.

2. Is the equation $y = 3x + 2$ parallel to $2y + 3x = 3$? Explain.

3. $l_1 \perp l_2$. If the slope of l_1 is $\dfrac{3}{4}$, and the slope of l_2 is $\dfrac{8}{x-6}$, find the value of x.

4. Find the slope of a line parallel to a line whose slope is $-\dfrac{2}{3}$.

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5. Find the slope of the line perpendicular to a line whose slope is $-\dfrac{2}{3}$.

6. Find the slope of a line parallel to the line whose equation is $3y + 2x = 6$.

7. Find the slope of a line perpendicular to the line whose equation is $3y + 2x = 6$.

8. Find the equation of the line parallel to the line whose equation is $y = -3x + 5$ and whose y-intercept is -5.

9. Find the equation of the line perpendicular to the line whose equation is $2y - 4x = 7$ and whose y-intercept is +5.

10. Find the equation of the line perpendicular to the line whose equation is $2y - 4x = 7$ and which passes through the point (1,2).

## 4. PARALLEL, COINCIDENT AND INTERSECTING LINES
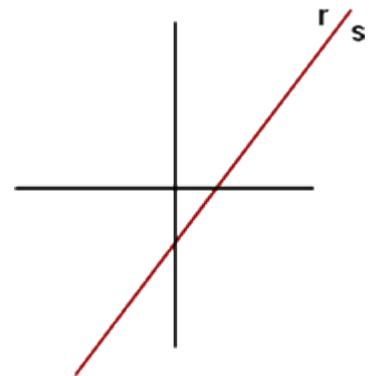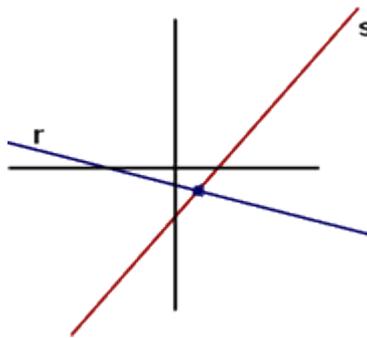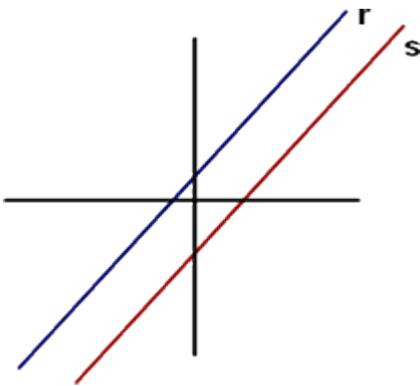
Parallel Lines are lines that never intersect. Two nonvertical lines are parallel if and only if they have the same slope.

Lines that intersect in a point are called **intersecting lines.**

Two lines that lie on top of one another are called **coincident lines.** Two straight lines are coincidental if all their points are common.

5. DISTANCE BETWEEN TWO POINTS

The distance between two points, A, and B, can be expresssed in terms of their coordinates by using the Pythagoras theorem. Let the coordinates of A, be (x_1, y_1) and let those of B be (x_2, y_2), as shown in figure:
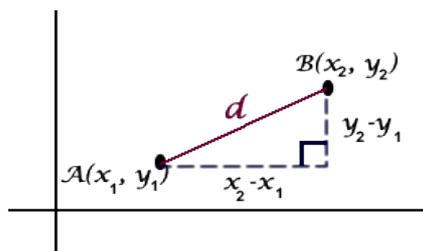
The **distance between two points** A=(x_1, y_1) and B=(x_2, y_2) in the plane is given by the formula:

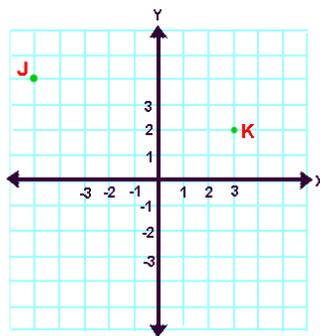$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

The distance between A=(-2,5) and B=(6,12) is given by:

$$d(A, B) = \sqrt{(6 + 2)^2 + (12 - 5)^2} =$$

$$= \sqrt{64 + 49} = \sqrt{113}$$

Exercises:

1. Find the length of the line segment whose endpoints are (-3, 4) and (5,4).

2. Find the distance between the points (-4,-5) and (1,-2).

3. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at (3,2)?

4. Find JK.

5. The coordinates of rectangle $ABCD$ are $A(0,2)$, $B(4,8)$, $C(7,6)$ and $D(3,0)$. Show that the diagonals are equal in length.

6. Two birds are flying toward a birdhouse that is halfway between them. The birds are at coordinates $A(-4,4)$ and $B(10,-2)$.

What are the coordinates of the birdhouse?

How far will each bird fly (to nearest tenth of a foot) to arrive at the birdhouse, if each grid on the graph represents 100 feet?
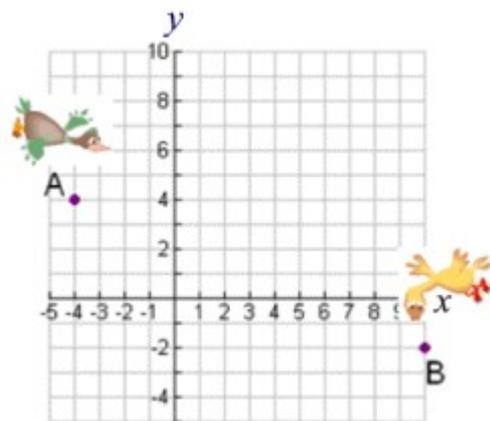
7. Given that $\overline{QU} \parallel \overline{DA}$.
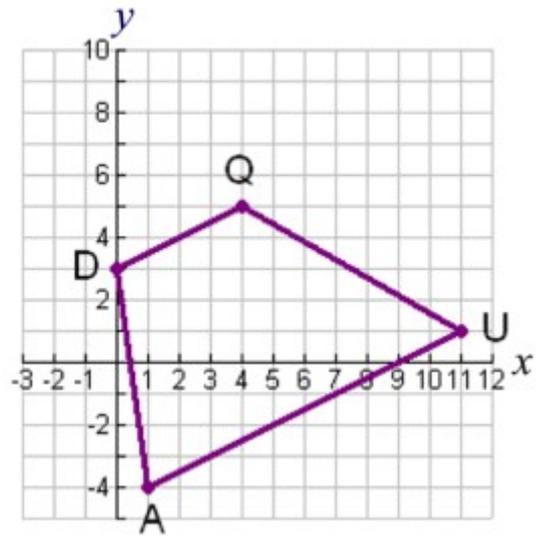
Q(4,5), U(11,1),
A(1,-4), D(0,3)

Find the slopes of $\overline{AU}$ and $\overline{DQ}$

Is *QUAD* a trapezoid?

Find the midpoints of $\overline{QU}$ and $\overline{DA}$ and label them *R* and *S*.

Find the lengths of $\overline{DQ}, \overline{RS}, \overline{AU}$

What is the relationship between these three segments?

Final exercises:

1. What is the slope of a line perpendicular to $2y = -6x - 10$?

Choose:

a) -3   b) 3   c) -1/3   d) 1/3

2. Given $3y - 4x = 5$ and $4y + 6 = 3x$, are these lines parallel, perpendicular or neither?
Choose:

a) parallel   b) perpendicular   c) neither

3. What is the equation of a line that passes through the point (4,-5) and is parallel to $3x + 2y = 12$?

Choose:

a) $y = -3x + 6$ b) $y = (3/2)x + 1$ c) $y = 3x + 1$ d) $y = (-3/2)x + 1$

4. What is the equation of a line that passes through the point (-1,-2) and is perpendicular to $-5x = 6y + 18$?

Choose:

a) $y = (6/5)x - (4/5)$ b) $y = (-6/5)x + (6/5)$ c) $y = (6/5)x + (4/5)$ d)$\bigcirc y = (-6/5)x - (4/5)$

5. Given $4y - 2x = 10$ and $-6y - 6 = -3x$, are these lines parallel, perpendicular or neither?
Choose:

a) parallel   b) perpendicular   c) neither

6. What is the equation of a line that is parallel to y = -4 and passes through the point (3,7)?

Choose:

a) $y = 3x - 4$ b) $y = 3$ c) $y = 3x + 7$ d) $y = 7$

7. Write the equation of a line that is parallel to 4x + 2y = -8 and has the same y-intercept as -3y = -2x - 9?
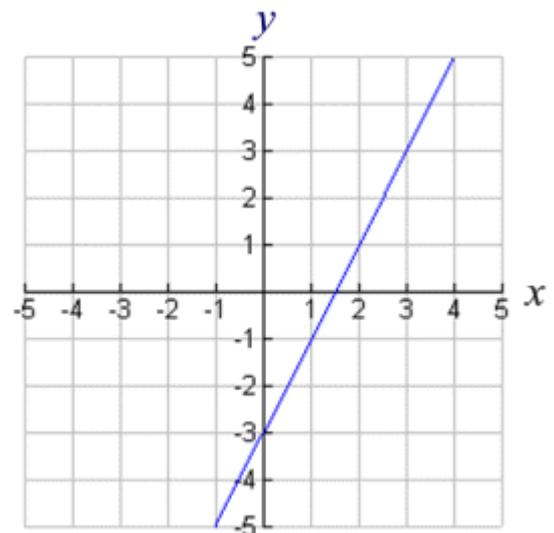
Choose:

a) $y = 4x - 9$ b) $y = -4x - 9$ c) $y = -2x - 3$ d) $y = -2x + 3$

8. A line passes through a point (2,5) and has a slope of -3. What is the equation of a line perpendicular to this line through (2,5)?

Choose:

a) $y = (1/3)x + 13/3$ b) $y = -3x + 17$ c) $y = -3x + 11$ d) $y = (1/3)x - 2/3$

9. What is te slope of a line that would be perpendicular to the line shown in this graph?
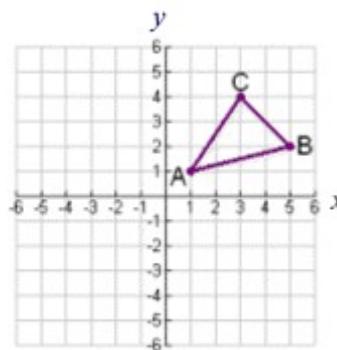
Choose:

a) -1/2 b) 2 c) 1/2 d) -2

10. Which of these equations represents a line parallel to the line 2x + y = 6?

Choose:

a) $y = 2x + 3$ b) $y - 2x = 4$ c) $2x - y = 8$ d) $y = -2x + 1$

11. Find the slope of each side of the triangle $A(1,1)$, $B(5,2)$, $C(3,4)$

12. A rancher is putting up a new fenced in area for his horses. The corners of the fenced area are located at $A(0,0)$, $B(20,0)$, $C(8,24)$ and $D(-8,16)$.

a. By checking slopes, show the relationship between $\overline{AD}$ and $\overline{BC}$.

b. By checking slopes, show the relationship between $\overline{AD}$, $\overline{BC}$ and $\overline{DC}$.

c. What kind of quadrilateral is the pasture, and why?